1. A beam *AB* has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points *C* and *D*, where AC = 1 m and DB = 1 m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end *B* as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at *D* is three times the magnitude of the reaction at *C*. By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end *B*.

(Total 7 marks)



A pole *AB* has length 3 m and weight *W* newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points *A* and *C* where AC = 1.8 m, as shown in the diagram above. A load of weight 20 N is attached to the rod at *B*. The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(a) Show that the tension in the rope attached to the pole at C is
$$\left(\frac{5}{6}W + \frac{100}{3}\right)N$$
. (4)

(b) Find, in terms of *W*, the tension in the rope attached to the pole at *A*.

(3)

Given that the tension in the rope attached to the pole at C is eight times the tension in the rope attached to the pole at A,

(c) find the value of *W*.

(3) (Total 10 marks)

2.

3.



A beam *AB* has mass 12 kg and length 5 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to *A*, the other to the point *C* on the beam, where BC = 1 m, as shown in the diagram above. The beam is modelled as a uniform rod, and the ropes as light strings.

- (a) Find
 - (i) the tension in the rope at C,
 - (ii) the tension in the rope at A.

(5)

(3)

A small load of mass 16 kg is attached to the beam at a point which is *y* metres from *A*. The load is modelled as a particle. Given that the beam remains in equilibrium in a horizontal position,

(b) find, in terms of y, an expression for the tension in the rope at C.

The rope at C will break if its tension exceeds 98 N. The rope at A cannot break.

(c) Find the range of possible positions on the beam where the load can be attached without the rope at *C* breaking.

(3) (Total 11 marks)





A uniform rod *AB* has length 1.5 m and mass 8 kg. A particle of mass *m* kg is attached to the rod at *B*. The rod is supported at the point *C*, where AC = 0.9 m, and the system is in equilibrium with *AB* horizontal, as shown in the diagram above.

(a) Show that m = 2.

A particle of mass 5 kg is now attached to the rod at A and the support is moved from C to a point D of the rod. The system, including both particles, is again in equilibrium with AB horizontal.

(b) Find the distance *AD*.

(5) (Total 9 marks)

(4)

5.



A uniform plank *AB* has weight 120 N and length 3 m. The plank rests horizontally in equilibrium on two smooth supports *C* and *D*, where AC = 1 m and CD = x m, as shown in the diagram above. The reaction of the support on the plank at *D* has magnitude 80 N. Modelling the plank as a rod,

(a) show that x = 0.75

(3)

A rock is now placed at *B* and the plank is on the point of tilting about *D*. Modelling the rock as a particle, find

(b) the weight of the rock,

Edexcel

(4)

(c) the magnitude of the reaction of the support on the plank at *D*.

(2)

(d) State how you have used the model of the rock as a particle.

(1) (Total 10 marks)

6.



A steel girder *AB* has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end *A*. The other cable is attached to the point *C* on the girder, where AC = 90 cm, as shown in the figure above. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at C is twice the tension in the cable at A, find

(b) show that AB = 120 cm.

A small load of weight W newtons is attached to the girder at B. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at C is now three times the tension in the cable at A.

(c) Find the value of *W*.

(7) (Total 13 marks)

(2)

(4)





A seesaw in a playground consists of a beam AB of length 4 m which is supported by a smooth pivot at its centre C. Jill has mass 25 kg and sits on the end A. David has mass 40 kg and sits at a distance x metres from C, as shown in the figure above. The beam is initially modelled as a uniform rod. Using this model,

(a) find the value of x for which the seesaw can rest in equilibrium in a horizontal position.

(3)

(b) State what is implied by the modelling assumption that the beam is uniform.

(1)

David realises that the beam is not uniform as he finds that he must sit at a distance 1.4 m from C for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a non-uniform rod of mass 15 kg. Using this model,

(c) find the distance of the centre of mass of the beam from *C*.

(4) (Total 8 marks)

8.



A uniform beam *AB* has mass 12 kg and length 3 m. The beam rests in equilibrium in a horizontal position, resting on two smooth supports. One support is at end *A*, the other at a point *C* on the beam, where BC = 1 m, as shown in the diagram. The beam is modelled as a uniform rod.

(a) Find the reaction on the beam at *C*.

(3)

A woman of mass 48 kg stands on the beam at the point D. The beam remains in equilibrium. The reactions on the beam at A and C are now equal.

(b) Find the distance *AD*.

(7) (Total 10 marks)

9.



A plank AB has mass 40 kg and length 3 m. A load of mass 20 kg is attached to the plank at B. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes attached at A and C, as shown in the diagram. The plank is modelled as a uniform rod and the load as a particle. Given that the tension in the rope at C is three times the tension in the rope at A, calculate

- (a) the tension in the rope at C,
- (b) the distance *CB*.

(5) (Total 7 marks)



10.

A plank of wood AB has mass 10 kg and length 4 m. It rests in a horizontal position on two smooth supports. One support is at the end A. The other is at the point C, 0.4 m from B, as shown in the diagram above. A girl of mass 30 kg stands at B with the plank in equilibrium. By modelling the plank as a uniform rod and the girl as a particle,

(a) find the reaction on the plank at *A*.

The girl gets off the plank. A boulder of mass m kg is placed on the plank at A and a man of mass 80 kg stands on the plank at B. The plank remains in equilibrium and is on the point of tilting about C. By modelling the plank again as a uniform rod, and the man and the boulder as particles,

(b) find the value of *m*.

(4) (Total 8 marks)

(4)





A plank AE, of length 6 m and mass 10 kg, rests in a horizontal position on supports at B and D, where AB = 1 m and DE = 2 m. A child of mass 20 kg stands at C, the mid-point of BD, as shown in the diagram above. The child is modelled as a particle and the plank as a uniform rod. The child and the plank are in equilibrium. Calculate

(a) the magnitude of the force exerted by the support on the plank at *B*,

(4)

(b) the magnitude of the force exerted by the support on the plank at *D*.

(3)

The child now stands at a point F on the plank. The plank is in equilibrium and on the point of tilting about D.

(c) Calculate the distance *DF*.

(4) (Total 11 marks)

12.



A lever consists of a uniform steel rod AB, of weight 100 N and length 2 m, which rests on a small smooth pivot at a point C of the rod. A load of weight 2200 N is suspended from the end B of the rod by a rope. The lever is held in equilibrium in a horizontal position by a vertical force applied at the end A, as shown in the diagram above. The rope is modelled as a light string.

Given that BC = 0.2 m,

(a) find the magnitude of the force applied at *A*.

(4)

The position of the pivot is changed so that the rod remains in equilibrium when the force at A has magnitude 120 N.

(b) Find, to the nearest cm, the new distance of the pivot from *B*.

(5) (Total 9 marks) 13.



A non-uniform rod *AB* has length 5 m and weight 200 N. The rod rests horizontally in equilibrium on two smooth supports *C* and *D*, where AC = 1.5 m and DB = 1 m, as shown in the diagram above. The centre of mass of *AB* is *x* metres from *A*. A particle of weight *W* newtons is placed on the rod at *A*. The rod remains in equilibrium and the magnitude of the reaction of *C* on the rod is 160 N.

(a) Show that 50x - W = 100.

The particle is now removed from A and placed on the rod at B. The rod remains in equilibrium and the reaction of C on the rod now has magnitude 50 N.

- (b) Obtain another equation connecting *W* and *x*.
- (c) Calculate the value of *x* and the value of *W*.

(4) (Total 12 marks)

(5)

(3)

14.



Edexcel

A uniform plank *AB* has mass 40 kg and length 4 m. It is supported in a horizontal position by two smooth pivots, one at the end *A*, the other at the point *C* of the plank where AC = 3 m, as shown in the diagram above. A man of mass 80 kg stands on the plank which remains in equilibrium. The magnitudes of the reactions at the two pivots are each equal to *R* newtons. By modelling the plank as a rod and the man as a particle, find

- (a) the value of R,
- (b) the distance of the man from *A*.

Moments - Moments about a point

(4) (Total 6 marks)

(2)

15.



- 2 m -

3 m-

D

B

(a) Show that
$$W = \frac{60}{1-x}$$

(b) Hence deduce the range of possible values of *x*.

0.5 m

(2) (Total 10 marks)

(8)



A plank AB has length 4 m. It lies on a horizontal platform, with the end A lying on the platform and the end B projecting over the edge, as shown above. The edge of the platform is at the point C.

Jack and Jill are experimenting with the plank. Jack has mass 40 kg and Jill has mass 25 kg. They discover that, if Jack stands at *B* and Jill stands at *A* and BC = 1.6 m, the plank is in equilibrium and on the point of tilting about *C*. By modelling the plank as a uniform rod, and Jack and Jill as particles,

(a) Find the mass of the plank.

They now alter the position of the plank in relation to the platform so that, when Jill stands at B and Jack stands at A, the plank is again in equilibrium and on the point of tilting about C.

(b) Find the distance *BC* in this position.

(5)

(3)

- (c) State how you have used the modelling assumptions that
 - (i) the plank is uniform,
 - (ii) the plank is a rod,
 - (iii) Jack and Jill are particles.

(3) (Total 11 marks)



[10]

3. (a)

$$S \longrightarrow C B$$

$$M(A): T \times 4 = 12g \times 2.5$$

$$T = 7.5g \text{ or } 73.5 \text{ N}$$

$$R(\uparrow) S + T = 12g$$

$$\Rightarrow S = 4.5g \text{ or } 44.1 \text{ N}$$

$$M1A1$$

$$A1$$

$$A1$$

$$A1$$

$$S = 4.5g \text{ or } 44.1 \text{ N}$$

(b)

$$U + V + V + C = \frac{16g}{16g} + \frac{12g}{12g} + \frac{12g}{16g} + \frac{12g}{12g} + \frac{12g}{12g} + \frac{12g}{16g} + \frac{12g}{12g} + \frac{12g}{12g}$$

(c)
$$V \le 98 \Rightarrow 39.2y + 73.5 \le 98$$

 $\Rightarrow y \le 0.625 = 5/8$ DM1
Hence "loads must be no more than 5/8 m from A" (o.e.) A1 3

[11]

4.	(a)	$M(C) \ 8g \times (0.9 - 0.75) = mg(1.5 - 0.9)$		M1A1	
		Solving to $m = 2 *$	CSO	DM1A1	4

(b)

$$A D B$$

 $5g \sqrt{x} \sqrt{x} \sqrt{2g} \sqrt{2g} \sqrt{M(D)} 5g \times x = 8g \times (0.75 - x) + 2g(1.5 - x)$
Solving to $x = 0.6$ ($AD = 0.6$ m)
M1A2(1,0)
DM1A1 5
[9]

5. (a) $M(C) \ 80 \times x = 120 \times 0.5$ M1A1 x = 0.75 *3 A1cso Using reaction at C = 0(b) **B**1 M(D) 120 × 0.25 = W × 1.25 ft their x M1A1 W = 24 (N) A1 4 (c) i X = 24 + 120 = 144 (N) ft their W M1A1ft 2

(d) The weight of the rock acts precisely at *B*. B1 1 [10]

4

7

6. (a)

$$A \xrightarrow{R} d \qquad \uparrow^{2R} \\ \downarrow^{210}$$

 $R + 2R = 210 \Longrightarrow R = 70 \text{ N}$ 2 A1 Note that they can take moments legitimately about many points for a valid method to get R (almost always resolving!)

(b) e.g. M(A): $140 \times 90 = 210 \times d$ A1ft

$$\Rightarrow d = 60 \Rightarrow AB = 120 \text{ cm}$$
A1

$$I^{st} \quad \text{for a valid moments equation}$$
A1

$$2^{nd} ext{ for complete solution to find AB (or verification)} \\ Allow 'verification', e.g. showing 140 × 90 = 210 × 60 ext{ A1} \\ 1260 = 1260 ext{ QED } ext{ A1} \\ \end{cases}$$

(c)

$$\begin{array}{c} & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & &$$

$$4S = 210 + W$$
e.g. M(B): $S \times 120 + 3S \times 30 = 210 \times 60$
Solve $\rightarrow (S = 60 \text{ and}) W = 30$
A1

Solve \rightarrow (*S* = 60 and) *W* = 30

In both equations, allow whatever they think S is in their equations for full marks(e.g. if using S = 70).

 2^{nd} A2 is for a moments equation (which may be about any one of 4+ points!)

 1^{st} A1 is for a second equation (resolving or moments) If they have two moments equations, given A2 if possible for the best one 2 M marks only available without using S = 70. If take mass as 210 (hence use 210g) consistently: treat as MR, i.e. deduct up to two A marks and treat rest as f.t. (Answers all as given = 9.8). But allow full marks in (b) (g's should all cancel and give correct result).

[13]

7. (a) M(C):
$$25g \times 2 = 40g \times x$$
 A1
 $x = 1.25 \text{ m}$ A1 3

(b) Weight/mass acts at mid-point; or weight/mass evenly distributed (o.e.) B1 1

(c)
$$y = 1.4$$
 M (C): $40g \times 1.4$
 $25 g = 15 g + 40 g$ A1
 $= 1.5g \times y + 25g \times 2$ A1
Solve: $y = 0.4$ m A1 4
[8]

8. (a)
$$\begin{array}{c} & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & &$$

(b)
$$S \uparrow x \to \uparrow S \to 12g$$

$$R(\uparrow) 2S = 48g + 12g$$

$$S = 30g$$

$$M(A): S \times 2 = 12g \times 1.5 + 48g \times x$$

$$Sub \text{ for } S \text{ and solve for } x: x = \frac{7/8 \text{ or } 0.875 \text{ or } 0.88 \text{ m}}{A1}$$

9.
$$T + 40g + 20g$$

(a) $R(\uparrow): T + 3T = 40g + 20g$
 $T = 15g$, so tension at C is $45g$ or 441 N or 440 N A1 2
(b) $M(B) = 15g \times 3 + 45g \times d = 40g \times 1.5$
Solve: $d = 1/3$ or 0.33 or 0.333 m A1 5
[7]









- (a) $R(\uparrow): 2R = 80g + 40g$ R = 60g or 588 N
- (b) M (A): $80g \times x + 40g \times 2 = 60g \times 3$ (-1 eeoo)

$$\Rightarrow x = \frac{5}{4} m \qquad \qquad A1 \qquad 4$$

[Allow moments eqn about any pt, but A2 for the eqn needs dist in terms of required 'x']

[6]

A1

A2 ft

2

(b)
$$W > 0 \Rightarrow x < 1$$
 A1 2 [10]

16. (a)

(b)

- 1. This question was well answered, particularly by those who resolved vertically to produce one of their equations. Those who took moments about two different points had a higher failure rate, partly because of the need to represent more lengths in terms of x and partly because of the heavier algebra required. Most had the R and 3R the right way round, and few were tempted to swap over Tom and Sophie. There were seven significant points on the beam, and the candidates between them took moments about all seven. The least successful seemed to be those who took moments about Tom's position, which generally led to errors in the distances. A few took moments about a point but equated the sum of the moments to the reaction at the point producing a dimensionally incorrect equation and losing all the marks for that equation. It was rare to see g's being used.
- 2. This question was well done by the majority of candidates and was the next best answered question after 1 and 2. Most made valid attempts at taking moments, in part (a) about A and often also about C in part (b). The printed answer was an additional help to the less able students who were able to score the marks in part (b) by using it in a vertical resolution. There was some confusion in the last part over the interpretation and use of the information given. Correct statements of simply Y = 8X or else 8X + X = W + 20 were seen but also X = 8Y was common as were the more surprising X + 8Y = W + 20 and 8X + Y = W + 20, both of which scored nothing.
- 3. Apart from the minority of candidates who, in their moments equation, failed to multiply the tension by a length (dimensionally incorrect \Rightarrow no marks) or those who omitted g, part (a) was well-answered. In the second part a number of candidates failed to re-arrange their moments equation to give an expression for the tension and made y the subject instead. Most proceeded, in the final part, via an equation rather than an inequality and very few made the final verbal statement referring specifically to the positioning of the load rather than a defined 'y'.
- 4. (a) Most candidates answered this correctly, usually taking moments about C, although a small minority took moments about A or D having first ascertained the normal reaction at C.
 - (b) Candidates were less successful with this part. The successful answers usually took moments about *D* which they placed to the left of the centre of mass and called the distance *AD* 'x'. This method obviated the need to find the normal reaction at *D*. Among those others who were also successful, the majority took moments about *A* having ascertained the normal reaction at *D* and again calling *AD* 'x'. Some candidates created three unknowns: *AD*, *DC* and *DB*; these candidates were rarely successful in their answers, succumbing to the difficulty of unravelling a complexity of their own making. Other candidates failed for various reasons: some for incorrectly calculating the normal because they missed out the mass of the rod or one of the other masses, more usually the former, others because when they took moments about *D* they failed to take account of the mass of the rod, more usually, or one of the other masses. Some candidates were unsuccessful because they placed *D* on the right of the centre of mass and then ran into problems using (7.5 x) rather than (x 7.5).

- 5. Many candidates found this question difficult, particularly parts (b) and (c). Candidates must be encouraged to state clearly where they are taking moments about and to make use of clearly labelled diagrams, showing relevant forces and distances. In part (a), most candidates took moments about *C* but a significant number resolved vertically and then took moments about *A* to obtain the printed answer. Many candidates, in the second part, did not realise that the reaction at C was now zero, and this meant that they made little progress. The introduction of g was also common and several candidates found the mass of the rock instead of its weight. Candidates who used a new diagram in part (b) and took care to include distances seemed to be more successful than others. Many were able to pick up a follow through mark in part (c) for resolving vertically correctly, using an incorrect value for the weight of the rock. The final part produced a full range of answers, many of which missed the point.
- 6. Although this was well done by many, it appeared to provoke a lot of crossing out with often multiple attempts made to parts (b) and (c), with working then continued in the space for other questions. Most could do part (a) successfully. In part (b), a 'forwards' approach, taking moments, was adopted by many, though a number adopted a verification ('backwards') approach, showing that two moments were equal if the distance was as given. In part (c) a number of correct solutions were seen; the common error among weaker candidates was to assume that the answer gained in part (a) still applied to this new situation: this then required only one equation in one unknown and considerably shortened the work required.
- 7. Again most candidates scored highly on this question, clearly understanding the principles of moments and applying them correctly. There were some arithmetical slips, but most could make good progress in the question. The statement required about the uniformity of the rod was generally answered in such as way as to gain the mark, though 'explanations' were at times a little unclear (e.g. that the weight was 'at' the middle of the rod, rather than acting through the middle of the rod).
- 8. Part (a) was generally well done, though there were still some answers given attempting to equate forces with moments. In part (b), quite a number of fully correct solutions were seen, though sometimes with not the most economic approach to the problem: some took moments about two different points which led to a fairly messy set of simultaneous equations to have to solve. Taking moments once, and resolving vertically, led to a much quicker solution! For those who went wrong (apart from making small slips processing equations), the most common error was to assume that the reactions in (b) had the same value as that found in (a).
- 9. In part (a) most could make a reasonable attempt at the question, though several effectively found the tension at A rather than at C, failing to multiply their answer by 3. In part (b) the general principle of taking moments was well known, and only a few candidates omitted forces (e.g. the weight) in their equations. Many fully correct answers were seen. A number of candidates still persist in failing to distinguish properly between weight and mass, omitting factors of g in their forces.

- 10. Part (a) was generally well done with most realising that they had to take moments. There was though some confusion at times between weights and masses. In part (b), several failed to appreciate what the statement about being 'on the point of tilting' meant in terms of the mechanics (i.e. that reaction at *A* was zero) and hence could make little progress with a viable solution; several also used their answer from part (a), failing to realise that the situation had changed significantly in terms of the forces.
- 11. Parts (a) and (b) were generally well done. Nearly all realised that they had to take moments and also fully appreciated that a moment was force x distance. Many correct solutions were seen though quite a number of candidates lost a mark through giving their answers to an inappropriate degree of accuracy with 4 or more significant figures. Part (c) caused more problems for many, with many assuming that the values of the reactions were unchanged from the previous parts of the question, others assuming that the reaction at *D* (rather than *B*) was zero.
- **12.** Again this proved to be reasonably accessible as a question with again a good number for fully correct solutions seen. Virtually all realise that they had to take moments, and nearly all included all the relevant forces in their equations, which was pleasing to see. The solution of the equations was also generally well done.
- **13.** Most realised that they had to take moments in the question; however, many failed to see that taking moments about *D* was by far the most economical way to approach the question. Some realised that they needed another equation and thus landed themselves with more complex algebra, and some simply ignored the reaction at *D* completely. In part (b), some failed to realise that the reaction at *C* had changed. But those who did succeed in writing down the two moments equations correctly usually managed to finish the question successfully, processing and solving the simultaneous equations accurately.
- 14. Generally this question was well answered with most candidates showing that they understood the relevant principles involved. Virtually all realised that they had to take moments at least once to solve the problem eventually, though some failed to see that the could do part (a) most easily by resolving. There was sometimes some confusion between weight and mass, with factors of g sometimes omitted. Also some candidates misread the data of the question and assumed that the combined reaction at the two pivots was R. However, there were also several fully correct solutions seen here.

- **15.** Most realised that they had to write down two equations and that at least one of them required taking moments. However, several made life more complicated for themselves than necessary by not resolving for their second equation. Some weaker candidates omitted the unknown weight *W* completely when resolving and hence thought that they could calculate the reactions numerically. Attempts to eliminate one unknown from the equations and find a relationship between *W* and *x* were rather variable in quality, with several fudges at the end of the working to produce the given answer. Reasoning in part (b) was generally almost non-existent. A variety of inequalities appeared but seldom with any justification.
- **16.** No Report available for this question.